An extended variant of the input-output model

Trinh Bui¹, Thai Nguyen Quang²

1. Researcher at Vietnam development research institute, correspondence author,
2. Director of Vietnam development research institute,

IJMSSSR 2021
VOLUME 3
ISSUE 6 NOVEMBER - DECEMBER

Abstract: Since the I/O model was created by W. Leontief, there have been many models extended from the I/O model such as the inter-regional I/O model, the inter-country I/O model, the social accounting matrix (SAM), demographic - economic model...The I/O model describes and analyzes the flow of products through inter-industry relationships or the economy's initial distribution process. The inter-region I/O model goes beyond the standard I/O model, it not only reflects the inter-sector relationship but also reflects the inter-regional relationship, it allows determining the advantage of one region over another in which sector? Other types of extended I/O such as the social accounting matrix and the demographic-economic model are not only described the initial distribution process, but also the distributive process of economy. This study expands the I/O model with a harmonious blend of the concept of the social accounting matrix and the demographic economic model to see that the economy is like a cycle of reincarnation, with no beginning and no end; any change (Cause) at any stage will directly or indirectly (Result) affect the rest.

Keywords: Extended, household, input, output, production, value added

Introductions

Since W. Leontief created the I/O model (1936, 1941), this model has been developed and expanded a lot. There have been many studies extending the original standard I/O model, such as Social Accounting Matrix-SAM (Richard Stone, 1961), System of National Accounts - SNA, Demographic-Economic Model (Miyazawa, 1968, 1971), the combined economic and environmental model (Hybrid Input-output table) and the interregional model (Miyazawa et al., 1976). These extended I/O models have been developed and applied by most countries in the world to analyze and forecast the economy (Pyatt and Roe, 1977; Cohen, 1988; Pyatt and Round, 1985, Bui et al, 2012). There are various uses on these models such as Input-output analysis, inter-regional, inter-country Input-output analysis, SAM analysis and CGE model. These analyzes are very diverse based on linear algebra, function analysis to flexibly transform Leontief's original standard model.

In this study, an extended I/O model is proposed with the number of columns opened such as: Household expenditure (includes final household consumption, personal income tax and saving), State expenditure (Government consumption, transfer and saving (investment), expenditure of the enterprise sector (gross capital formation, corporate income tax and property payment) and last column is Rest of the World. In this model, rows are also opened corresponding to the income of the household, the income of the State sector, the income of the enterprises sector and relationship with Rest of the World.

The table 1 below depicts the above idea. It not only depicts the effects of direct and indirect taxes, but also quantifies the effect of exports on the value added factors. In a previous study, Bui Trinh (2020) also introduced a method of estimating exports affecting supply-side factors; however, this study considers the influence of demand-side on output and factors of value added through the arrangement of the sub-matrix

Table 1. Extended I/O Model Diagram

<table>
<thead>
<tr>
<th>Intermediate input</th>
<th>Households</th>
<th>Government</th>
<th>Production</th>
<th>Rest of the World</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intermediate demand</td>
<td>X₁</td>
<td>X₂</td>
<td>X₃</td>
<td>X₄</td>
<td>F₁</td>
</tr>
<tr>
<td>Households</td>
<td>COE (H₁.X₁)</td>
<td>H₂.X₃</td>
<td>F₂</td>
<td>X₂</td>
<td></td>
</tr>
</tbody>
</table>
With:

+ $A^d$ is a coefficient matrix of domestic intermediate input; $X_{i1}$ is a vector of gross output;
+ $C^d$ is coefficient matrix (or vector) of domestic household final consumption; $X_2$ is a vector of total income of household;
+ $G^d$ is coefficient vector of domestic Government final consumption,
+ $I^d$ is coefficient vector of domestic gross capital formation; $X_3$ is a vector of total income of enterprises sector;
+ $H^1$ is coefficient of (or matrix) compensation of employees, $H_3$ is coefficient of transfer income by Government;
+ $T^1$ is the coefficient of State revenue from indirect taxes (excluding subsidies), $T^2$ is the personal income tax coefficient and $T^4$ is the corporate income tax that the State sector receives from households and the enterprises sector.
+ $S_1$ is the coefficient of operating surplus, $S_2$ is the coefficient of savings and property payment of the household sector, $S_3$ is the coefficient of saving (investment) of state area, $F_1$ is export vector, $F_2$ is other non-production income of the household sector, $F_3$ is non-tax revenue of the State sector and $F_4$ is other income of the enterprises sector.
+ $M_1$ is import for intermediate inputs, $M_2$ is imports and property payment of household sector, $M_3$ is imports and property payment of state sector, $M_4$ is import for gross capital formation and property payment of enterprises sector.

**Approach**

With the above diagram, we have the following relationships:

\[ \begin{align*}
A^d X_1 &+ C^d X_2 + G^d X_3 + I^d X_4 + F_1 = X_1 \\
H_1 X_1 &+ H_3 X_3 + F_2 = X_2 \\
T_1 X_1 &+ T_2 X_2 + T_3 X_4 + F_3 = X_3 \\
S_1 X_1 &+ S_2 X_2 + S_3 X_3 + F_4 = X_4
\end{align*} \]  

Relation (1) is the Leontief standard relation describing the product transaction flow with: $A^d X_1$ represents domestic intermediate consumption; $C^d X_2$ is a matrix (vector) of domestic household final consumption; $G^d X_3$ is a vector of domestic Government final consumption expenditure; $I^d X_4$ is domestic gross capital formation and $F_1$ is an export vector; $X_1$ is a gross output vector.

Relation (2) shows the total income of household $X_2$ including income from production $H_1 X_1$, transfer from the state sector $H_3 X_3$ and other income $F_2$.

Relation (3) represents the total revenue of the state $X_3$ including indirect tax $T_1 X_1$ personal income tax $T_2 X_2$, corporate income tax $T_3 X_4$ and other revenue $F_3$.

Relation (4) represents the total income of the business sector $X_4$ includes operating surplus $S_1 X_1$, savings of household area $S_2 X_2$, investment (saving) of state area $S_3 X_3$ and other income of business sector $F_4$.

The above diagram can also be seen written as follows:

\[ \begin{align*}
A^d X_1 &+ H_1 X_1 + T_1 X_1 + S_1 X_1 + M_1 = X_1
\end{align*} \]
\[ C^d X_2 + T_2 X_2 + S_2 X_2 + M_2 = X_2 \]  
\[ G^d X_3 + H_3 X_3 + S_3 X_3 + M_3 = X_3 \]  
\[ I^d X_4 + T_4 X_4 + M_4 = X_4 \]  

Relation (5) represents the supply side includes: Domestic intermediate input matrix \((A^d)\), import intermediate input vector (or matrix) \(M_1\) and values added \((H_1 X_1 + T_1 X_1 + S_1 X_1)\)

Relation (6) shows savings of the household sector \((S_2 X_2)\) equal total income \((X_2)\) minus final household consumption \((C^d X_2 + M_2)\) and personal income tax \((S_2 X_2)\)

Relation (7) represents the total expenditure of the State sector including final consumption expenditure of domestic products and imported products \((G^d X_3 + M_3)\), transfer expenditure for the household sector \((H_3 X_3)\) and expenditure for investment \((S_3 X_3)\)

Relation (8) represents the gross capital formation of the enterprises sector from domestic products and imported products \((I^d X_4 + M_4)\) and corporate income tax \((T_4 X_4)\).

Relations (1), (2), (3), (4) and (5), (6), (7), (8) can be rewritten as a matrix as follows:

\[
\begin{bmatrix}
A^d & C^d & G^d & I^d \\
H_1 & H_3 & & \\
T_1 & T_2 & T_4 & \\
S_1 & S_2 & S_3 & \\
\end{bmatrix}
\begin{bmatrix}
X_1 \\
X_2 \\
X_3 \\
X_4 \\
\end{bmatrix}
+ 
\begin{bmatrix}
F_1 \\
F_2 \\
F_3 \\
F_4 \\
\end{bmatrix}
= 
\begin{bmatrix}
X_1 \\
X_2 \\
X_3 \\
X_4 \\
\end{bmatrix}
\]

Put:

\[
B = 
\begin{bmatrix}
A^d & C^d & G^d & I^d \\
H_1 & H_3 & & \\
T_1 & T_2 & T_4 & \\
S_1 & S_2 & S_3 & \\
\end{bmatrix}
\]

\[
X = 
\begin{bmatrix}
X_1 \\
X_2 \\
X_3 \\
X_4 \\
\end{bmatrix}
\]

\[
F = 
\begin{bmatrix}
F_1 \\
F_2 \\
F_3 \\
F_4 \\
\end{bmatrix}
\]

So we have:

\[
BX + F = X
\]

Hence, the Leontief relation is satisfied:

\[
X = (I - B)^{-1} F
\]

Put \(U = (I - B)^{-1}\)

At that time \(U\) was performed:

\[
U = 
\begin{bmatrix}
U^A & U^C & U^G & U^I \\
U^H_1 & U^H_3 & & \\
U^T_1 & U^T_2 & U^T_4 & \\
U^S_1 & U^S_2 & U^S_3 & \\
\end{bmatrix}
\]

And:

\[
X = U.F
\]

Follow Michael Sonis and Geoffrey (1993), Bui, T (2020); \(U^A\) is called the extended Leontief inverse matrix, This
matrix can be analyzed explicitly including the direct effects \( A^d \), indirect effects \([I - A^d]^{-1} - A^d\) and effects induced by final household consumption, final consumption of Government and gross capital formation: \(U^A - (I - A^d)^{-1}\). So, the spillover effects of an export unit to the factors of value added is determined:

\[
V = \begin{bmatrix}
U^H \\
U^T \\
U^S
\end{bmatrix}
\]  

(14)

Explain that matrix \( V \) can be understood:
- \(U^H\) is a matrix (vector) that induced impact by an export unit to the compensation of employees
- \(U^T\) is a vector that induced impact by an export unit to indirect tax
- \(U^S\) is a matrix (vector) that induced impact by an export unit to operating surplus.

Here the matrix \( M \) is called the extended Miyazawa matrix:

\[
M = \begin{bmatrix}
U^H \\
U^T \\
U^S
\end{bmatrix}
\]  

(15)

And \( Q \) is the matrix of output induced by the factors of domestic final demand

\[
Q = \begin{bmatrix}
U^C \\
U^G \\
U^I
\end{bmatrix}
\]  

(16)

Based on Miyazawa, Sonis and Hewwing the matrix \( U \) is rewritten as below:

\[
U = \begin{bmatrix}
U^A & U^A.Q \\
V.(I - A^d)^{-1} & I + V.A^d.Q
\end{bmatrix}
\]  

(17)

In the equation (7) matrix \( U \) is decomposed to sub – matrices in order to explain clearly on multipliers in economy.

**Empirical study.**

In this study, the number of industries is grouped into 3 major industry groups:
- Sector group 1: Agriculture, forestry and fishery
- Sector group 2: Industry and construction,
- Sector group 3: Service industry group.

The diagram of the The diagram of the expanded I/O model for Vietnam in 2016 is as follows I/O model for Vietnam in 2016 is as follows

**Table 1. The expanded input – output model of Vietnam economy, 2016 (Million USD)**

<table>
<thead>
<tr>
<th>Sector group 1</th>
<th>Sector group 2</th>
<th>Sector group 3</th>
<th>Household institutional</th>
<th>Government institutional</th>
<th>Production institutional</th>
<th>Rest of the World</th>
<th>Total</th>
</tr>
</thead>
</table>

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Table 2 generally shows that the spillover from exports to output of all three sector groups is higher than the spillover from domestic final demand (final consumption and gross capital formation). Calculation results show that the spillover from export to output of sector group 2 is highest and the lowest to be sector group 3. In contrast, the spillover from domestic demand to the service sector group is highest, followed by sector group 1 and finally sector group 2.

Table 2. Induced impacts of export and domestic final demand to outputs (Times)

<table>
<thead>
<tr>
<th></th>
<th>Sector group 1</th>
<th>Sector group 2</th>
<th>Sector group 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Export to output</td>
<td>2.121</td>
<td>2.271</td>
<td>1.904</td>
</tr>
<tr>
<td>Domestic final demand to output</td>
<td>1.513</td>
<td>1.259</td>
<td>1.700</td>
</tr>
</tbody>
</table>

Table 3 shows that overall, household spending has the most spillover to the general income redistribution and spending by the institutional sectors also has the most spillover to household income.

The expenditure of the household sector induce to the most to the income from redistribution of the sector itself, then the production sector and finally the State sector.

Table 3. Relationship between expenditures and incomes of institutional areas (times)

<table>
<thead>
<tr>
<th></th>
<th>Household institutional</th>
<th>Government institutional</th>
<th>Production institutional</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Household institutional</td>
<td>0.481</td>
<td>0.395</td>
<td>0.278</td>
<td>1.153</td>
</tr>
<tr>
<td>Government institutional</td>
<td>0.160</td>
<td>0.136</td>
<td>0.099</td>
<td>0.394</td>
</tr>
<tr>
<td>Production institutional</td>
<td>0.370</td>
<td>0.309</td>
<td>0.223</td>
<td>0.902</td>
</tr>
<tr>
<td>Total</td>
<td>1.011</td>
<td>0.839</td>
<td>0.599</td>
<td></td>
</tr>
</tbody>
</table>

Recurrent spending and transfers of the State sector also have the strongest effect on the income from redistribution of the household sector, followed by the production sector and finally the State sector.
Corporate spending on investment and corporate income tax also spill over into household redistribution income, followed by the itself.

These may also be due to the structure of income from production accounting for 77% in the net gross value added at basic prices of the whole economy (This ratio of industry group 1 is 78%, industry group 2 is 78%, 78% and industry group 3 is 76%)  

Conclusions

The study shows that the basic input-output model can be applied flexibly with variations to extend this model depending on the research purpose.

The study shows that the model is consistent with reality when it shows that although exports have a strong impact on the production value of all three major industry groups, they have a low impact on the factors of added value, especially the lowest induce to budget revenue.

The study also shows that out of 3 main groups of industries, priority should be given to exporting services because this group of industries spreads best to added value. Thus, the export priority policy should prioritize the export of services, followed by the export of products of the agricultural, forestry and fishery sectors. Exporting manufactured products is actually just exporting for other countries because the manufacturing industry in Vietnam is basically outsourcing.

In addition, the calculation results can show that household spending spreads to the best redistributive process. This requires further study of the structure of workers' income from production in total net value added at basic prices. This ratio is too high, incompatible with labor productivity, which can lead to distortions affecting the investment incentives of the production sector.

Due to the lack of data, this research is not possible to break down the sector and institutional sectors in more detail. It is hoped that this study can serve as a suggestion so that when conditions permit the sectoral and institutional sectors can be further decomposed.

Acknowledgment: I would like to thank Mr. Quoc Bui and Mr. Keith Yin supported this research

References


